Accepted version of (Metroeconomica, paper #366)

Heuristic expectation formation and business cycles: A simple linear model*

Frank H. Westerhoff

University of Osnabrück

Department of Economics

Rolandstraße 8

D-49069 Osnabrück

Germany

email: frank.westerhoff@uos.de

Abstract

We develop a Keynesian business cycle model to study how extrapolative and regressive expectation formation rules may affect fluctuations in economic activity. We find that simple expectation formation rules may have an impact on the level and the stability of the equilibrium income, the size of the multiplier and the resulting adjustment process after an exogenous shocks. Our model also reveals that national income may be influenced by how agents perceive their long-run average income.

Keywords

business cycles, bounded rationality, heuristic expectation formation, learning

JEL classification

E12, E32, D84

* I thank two anonymous referees for helpful comments, interesting suggestions and patience.

1 Introduction

All modern industrial economies regularly experience significant short-run output variations. Since every recession in which workers become involuntarily unemployed results in a loss of output that cannot be regained, the origins of business cycles belong to the most challenging issues of macroeconomics. Business cycles may, of course, arise from exogenous supply side shocks to a basically stable economy. According to Keynesian models, however, fluctuations in economic activity may furthermore be driven by changes in aggregate demand due to the instability of consumer and investor sentiment. Moreover, Day and Shafer (1985), Franke and Lux (1993), Hommes (1995), and Puu, Gardini, and Sushko (2005) even show that business cycles may be completely endogenous when the underlying law of motion is nonlinear. As is well known, nonlinearities may lead to chaotic motion. Comprehensive surveys of this topic are provided by Day (1999) and Rosser (2000).

Note that when economic variables evolve chaotically, it may become quite difficult to form rational expectations (Rosser 1996). As argued by Heiner (1983), agents may then retreat to more simplistic expectation formation rules. In fact, there exists a huge amount of empirical evidence, ranging from survey studies to laboratory experiments, stating that agents typically rely on relatively simple heuristics when having to predict future economic variables (Kahneman, Slovic, and Tversky 1986, Smith 1991). Westerhoff (2006a, 2006b) therefore seeks to explore in how far the expectation formation of boundedly rational agents may affect the evolution of national income. Since the agents use a nonlinear mix of extrapolative and regressive predictors to forecast national income, quite complex business cycles dynamics may be observed. Due to nonlinearities, however, it is quite difficult to pin down the causalities acting inside these models.

The goal of our paper thus is to develop a simple linear business cycle model which allows us to study the interplay between different expectation formation rules and national income more precisely. The macroeconomic side of our model is represented by the multiplier model. But contrary to the assumption that current consumption expenditures are a function of the last period's income, the agents consume a given fraction of their current expected income. The agents either extrapolate past output changes into the future or expect that output returns towards some long-run average value. We also consider the case in which agents hold optimistic or pessimistic beliefs concerning their long-run average income. The fraction of agents who follow one or the other predictor is fixed (and thus the model remains linear). Overall, the model has the potential to produce quite interesting dynamics. We find, for instance, that heuristic expectation formation may stimulate fluctuations in economic activity and that national income may (at least temporarily) disconnect from its equilibrium level when the agents become optimistic or pessimistic.

The remainder of our paper is organized as follows: In section 2, we develop a Keynesian business cycle model in which agents form boundedly rational expectations. In section 3, we first focus on a special case and derive some analytical results. In Section 4, investment becomes random. In section 5, agents try to learn the long-run average income level. The last section concludes the paper.

2 The model

Within a Keynesian multiplier model, national income Y_t at time t may be written as the sum of two components: investment I_t and consumption C_t. Hence,

$$Y_t = I_t + C_t . \tag{1}$$

For simplicity, investment fluctuates around a constant level $\bar{I}$

$$I_t = \bar{I} + \delta_t, \tag{2}$$

where δ is a normally distributed random variable with mean zero and constant standard deviation σ^{δ}.

Consumption is a function of national income. Typically, one assumes that the agents consume a given fraction of their last period's income.[1] However, recent empirical evidence (Carroll, Fuhrer and Wilcox 1994, Souleles 2004 or Doms and Morin 2004) suggests that consumer expenditures are driven by consumer sentiment, thus confirming Keynes' suspicion that consumer "attitudes" and "animal spirits" may cause fluctuations in economic activity. In particular, Souleles (2004) finds that higher consumer confidence is correlated with less saving and increases in expected future income (which is consistent with precautionary motives but counter to the permanent income hypothesis). Within our model, the agents consumption expenditures thus depend on their current expected income $E[Y_t]$, i.e.

$$C_t = cE[Y_t]. \tag{3}$$

The consumption parameter c is restricted to $0 < c < 1$.

Similar to Asada et al. (2003) and Chiarella, Flaschel and Franke (2005), the agents make use of extrapolative and regressive forecasting rules. The average market expectation with respect to national income is defined as

$$E[Y_t] = wE^e[Y_t] + (1-w)E^r[Y_t], \tag{4}$$

where the relative weight of extrapolative expectations $E^e[Y_t]$ is denoted by w and the relative weight of regressive expectations $E^r[Y_t]$ is represented by $(1-w)$.

---

[1] For $C_t = cY_t$ and $I_t = \bar{I}$, national income evolves as $Y_t = \bar{I} + cY_{t-1}$. Equilibrium income $\bar{Y} = \bar{I}/(1-c)$ is always stable since the consumption parameter c is by definition smaller than one.

When the agents compute their expected income for period *t*, they possess information up to period *t*-1. Extrapolative expectations may be formalized as

$$E^e[Y_t] = Y_{t-1} + a'(Y_{t-1} - Y_{t-2}), \tag{5}$$

where $a' > 0$ indicates how strongly the agents extrapolate past output changes.

Agents who form regressive expectations believe that national income will revert to some long-run average value $\hat{Y}$. This "normal" income level $\hat{Y}$ may, of course, not necessarily be identical to the true equilibrium income $\overline{Y}$. Regressive expectations may be expressed as

$$E^r[Y_t] = Y_{t-1} + b'(\hat{Y}_t - Y_{t-1}). \tag{6}$$

The agents consequently expect the gap between $\hat{Y}_t$ and Y_{t-1} to be reduced by a factor $0 < b' < 1$.

Finally, we have to specify how agents perceive their long-run average income level. Here we assume that agents behave as econometricians but are also influenced by Keynesian "animal spirits". Therefore, we write

$$\hat{Y}_t = d\,\hat{Y}_{t-1} + (1-d)\,Y_{t-1} + \varepsilon_t. \tag{7}$$

Accordingly, agents form a weighted average of the previously perceived long-run average income level and the last observed income level with $0 \le d \le 1$. The random term ε, which may be regarded as "animal spirits", is normally distributed with mean zero and constant standard deviation σ^{ε}. Note that there is indeed empirical evidence (Carroll, Fuhrer and Wilcox 1994, Souleles 2004, Doms and Morin 2004) that consumer sentiment is affected by many factors, for instance, by announcements of policy makers, media reports or natural disasters. These factors may be unrelated to the actual condition of the economy.

3 A special case: The deterministic skeleton with *d* = 1

To gain some basic insight into the dynamics of the model, we first explore a special case. In the following, we focus on the deterministic skeleton of the model with $d = 1$. Defining $a = a'w$ and $b = b'(1-w)$ and combining (1)-(7) reveals that

$$Y_t + (cb - ca - c)Y_{t-1} + caY_{t-2} = \bar{I} + cb\hat{Y}, \tag{8}$$

i.e. the recurrence relation which determines national income is a two-dimensional deterministic linear difference equation.[2]

Next we determine the fixed point of the model, characterize its stability, and calculate under which conditions cyclical output fluctuations may occur. Inserting $\bar{Y} = Y_{t-1} = Y_{t-2}$ into (8) reveals that

$$\bar{Y} = \frac{\bar{I} + cb\hat{Y}}{1 + cb - c}. \tag{9}$$

The fixed point of our model is only equal to the well-know Keynesian multiplier solution $\bar{I}/(1-c)$ when $\bar{Y} = \hat{Y}$. Put differently, if the agents perceive a lower (higher) average income level, it will indeed be lower (higher) than the traditional multiplier solution. The size of the multiplier may also be affected by the agents' expectation formation. The stronger the impact of regressive expectations, the less strongly equilibrium income reacts to changes in autonomous expenditures $\bar{I}$. Due to $0 < cb < c$, however, the multiplier always remains larger than one.

Let us now turn to the stability of the fixed point. Remember that a two-dimensional linear difference equation $X_{t+1} + a_1 X_t + a_2 X_{t-1} = Z$ is stable if (i) $1 + a_1 + a_2 > 0$, (ii) $1 - a_1 + a_2 > 0$, and (iii) $1 - a_2 > 0$. For our model, this is true if the three inequalities

$$1 + cb - c > 0, \quad (10)$$

$$1 + c + 2ca - cb > 0 \quad (11)$$

and

$$c < 1/a \quad (12)$$

hold. Obviously, only the last inequality imposes a real restriction on the stability of the fixed point. Interestingly, it is independent of the regressive forecasting rule.

Furthermore, the condition that a two-dimensional linear difference equation generates cycles is $a_1^2 < 4a_2$. For our model, this results in

$$c < 4a/(b - a - 1)^2, \quad (13)$$

which now depends on both forecasting rules.

Conditions (12) and (13) are plotted in figure 1 in (c, a) space for $b \in \{0, 0.15, 0.3, 0.45\}$. The four panels also contain the restrictions $a > 0$ and $0 < c < 1$. Parameter combinations which are located in the area marked with a "Z" produce cyclical output movements with decreasing amplitude. Note that the area which generates dampened oscillations increases with b.

---------- Figure 1 goes about here ----------

The three panels of figure 2 show the evolution of national income after a one percent output shock in period $t = 1$ for 100 time steps. The underlying parameter setting is given as $a = a'w = 1.25 * 0.8 = 1$, $b = b'(1 - w) = 0.25 * 0.2 = 0.05$, $c = 0.9$ and $\bar{I} = 10$. The three panels differ with regard to how the agents perceive their long-run average income. From top to bottom, we assume $\hat{Y} = 100$, $\hat{Y} = 102$ and $\hat{Y} = 98$, respectively. As a result, the true equilibrium income is given as $\bar{Y} = 100$, $\bar{Y} = 100.62$

---

[2] Our model is formally similar to Samuelson's (1939) famous multiplier-accelerator model.

and $\overline{Y} = 99.38$, respectively. All three scenarios display dampened output oscillations, i.e. output converges in the long run towards its steady-state value with decreasing amplitude. The cyclical behavior of the national income variable is, of course, triggered by the agents' boundedly rational expectation formation.

---------- Figure 2 goes about here ----------

4 Random investment

Note that agents who rely on regressive expectations may commit trivial forecast errors in equilibrium (see, e.g., the bottom two panels of figure 2). Within a pure deterministic setting, this seems not to be very appealing. However, we are living in a stochastic world where random shocks permanently enter the picture. Figure 3 therefore presents the same simulation run as in figure 2 (now for 200 time steps), except that $\delta \sim N(0,0.1)$. Obviously, the dynamics becomes much more realistic. Neither the amplitude nor the frequency of the cycles look very regular. Since the system does not converge to a fixed point, the agents may not necessarily (quickly) recognize that they misperceive the true equilibrium value of national income.

---------- Figure 3 goes about here ----------

5 Random investment and learning

Finally, we are ready to explore how the perception of the "normal" income level may affect the evolution of national income. The design of figure 4 is as in the bottom panel of figure 3 but now we set from top to bottom $d = 1, \sigma^{\varepsilon} = 0$, $d = 0.99, \sigma^{\varepsilon} = 0$, $d = 0.95, \sigma^{\varepsilon} = 0$, $d = 0.99\ \sigma^{\varepsilon} = 0.1$ and $d = 0.95, \sigma^{\varepsilon} = 0.1$, respectively. The thick gray line represents the perceived long-run average income level. In the first panel, agents underestimate their "normal" income level. In the following two panels, they seek to learn $\hat{Y}$. And in fact, for $d = 0.95$, the agents quickly come up with a reasonable guess of $\hat{Y}$. On the other hand, the bottom two panels show that “animal spirits” may at least temporarily lead to larger misperceptions. As can be seen, the perception of the “normal” income level has a market influence on the actual income level. Compare, for instance, the first and the last panel between period 40 to 120. In the top panel, the agents underestimate $\hat{Y}$ and thus actual income is relatively low. In the bottom panel, $\hat{Y}$ is overestimated and now actual income is much higher.

--------- Figure 4 goes about here ----------

6 Conclusions

Fluctuations in economic activity may be driven by numerous forces. The goal of the present paper is to clarify the role of some widely used expectation formation rules within a Keynesian business cycle model. The agents may use extrapolative and regressive prediction rules to forecast their income. Since the fractions of both predictors are fixed, the model is linear. We find that the equilibrium income is influenced by the agents’ expectations. If they become pessimistic (optimistic), equilibrium income decreases (increases). Moreover, the stability of equilibrium income decreases with the intensity of extrapolative expectations whereas the size of the multiplier decreases with the intensity of regressive expectations. Compared to the classical multiplier model, dampened oscillations may occur. Stochastic versions of the model generate even more interesting dynamics.

References

Asada, T., Chiarella, C., Flaschel, P. and Franke, R. (2003): Open Economy Macrodynamics: An Integrated Disequilibrium Approach. Springer: Berlin

Carroll, C., Fuhrer, J. and Wilcox, D. (1994): Does consumer sentiment forecast household spending? If so, why? American Economic Review, 84, 1397-1408.

Chiarella, C., Flaschel, P. and Franke, R. (2005): Foundations for a Disequilibrium Theory of the Business Cycle: Qualitative Analysis and Quantitative Assessment. Cambridge University Press: Cambridge.

Day, R. and Shafer, W. (1985): Keynesian chaos. Journal of Macroeconomics, 7, 277-295.

Day, R. (1999): Complex Economic Dynamics: An Introduction to Macroeconomic Dynamics (Volume 2). MIT Press: Cambridge.

Doms, M. and Morin, N. (2004): Consumer sentiment, the economy, and the news media. Federal Reserve Bank of San Francisco, Working Paper 2004-09.

Franke, R. and Lux, T. (1993): Adaptive expectations and perfect foresight in a nonlinear Metzlerian model of the inventory cycle. Scandinavian Journal of Economics, 95, 355-363.

Heiner, R. (1983): The origin of predictable behavior. American Economic Review, 73, 560-595.

Hommes, C. (1995): A reconsideration of Hicks' nonlinear trade cycle model. Structural Change and Economic Dynamics, 6, 435-459.

Puu, T., Gardini, L. and Sushko, I. (2005): A multiplier-accelerator model with floor determined by capital stock. Journal of Economic Behavior and Organization 56, 331-348.

Rosser, J. B. (1996): Chaos theory and rationality in economics. In: Kiel, D and Elliott, E. (eds.): Chaos Theory in the Social Sciences: Foundations and Applications, Michigan University Press: Ann Arbor, 199-213.

Rosser, J. B. (2000): From Catastrophe to Chaos: A General Theory of Economic Discontinuities (Second Edition). Kluwer Academic Publishers: Boston.

Samuelson, P. (1939): Interactions between the multiplier analysis and the principle of acceleration. Review of Economic Statistics, 21, 75-78.

Smith, V. (1991): Papers in Experimental Economics. Cambridge University Press: Cambridge.

Souleles, N. (2004): Expectations, heterogeneous forecast errors, and consumption: Micro evidence from the Michigan Consumer Sentiment Survey. Journal of Money, Credit and Banking, 36, 39-72.

Kahneman, D., Slovic, P. and Tversky, A. (1986): Judgment under Uncertainty: Heuristics and Biases. Cambridge University Press: Cambridge.

Westerhoff, F. (2006a): Samuelson's multiplier-accelerator model revisited. Applied Economics Letters, 13, 89-92.

Westerhoff, F. (2006b): Business cycles, heuristic expectation formation and contracyclical policies. Journal of Public Economic Theory (in press).

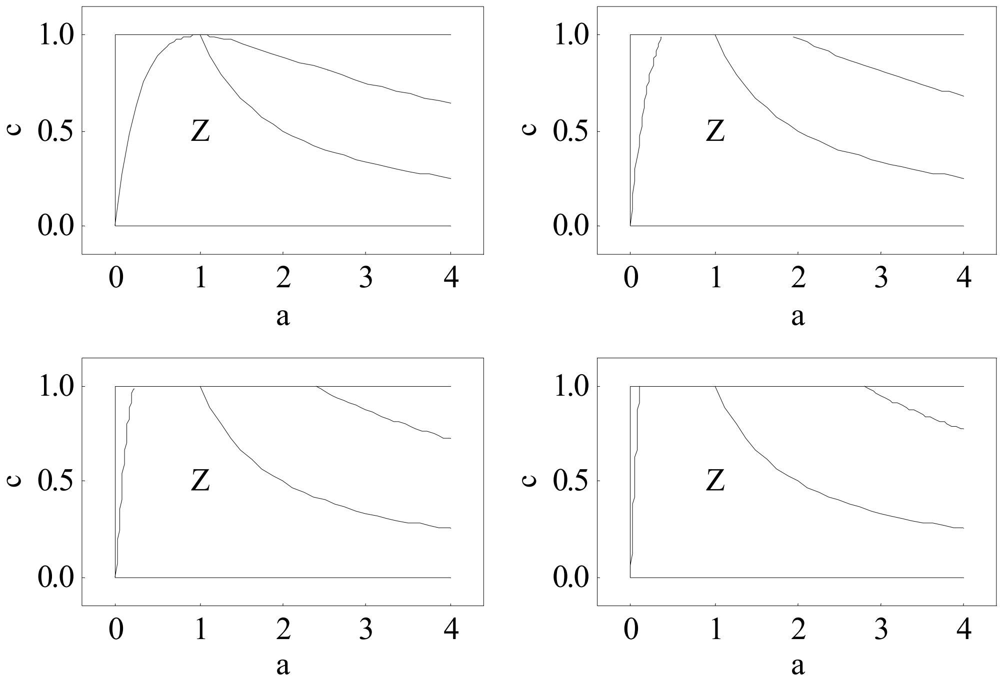

Figure 1: Conditions for stability and cycles in (c, a) space for $b \in \{0, 0.15, 0.3, 0.45\}$. Parameter combinations which produce dampened output oscillation are located in the area marked with a "Z". Remember that $a > 0$ and $0 < c < 1$.

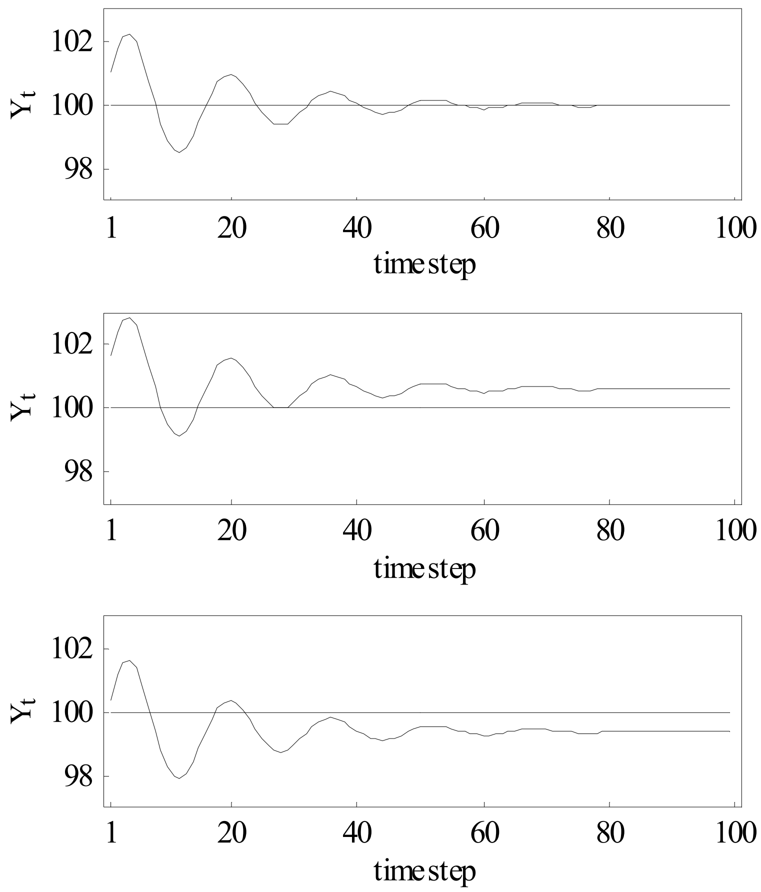

Figure 2: National income after a one percent output shock. From top to bottom we set $\hat{Y} = 100$, $\hat{Y} = 102$ and $\hat{Y} = 98$, respectively. The remaining parameters are given as $a = a'w = 1.25 * 0.8 = 1$, $b = b'(1-w) = 0.25 * 0.2 = 0.05$, $c = 0.9$, and $\bar{I} = 10$.

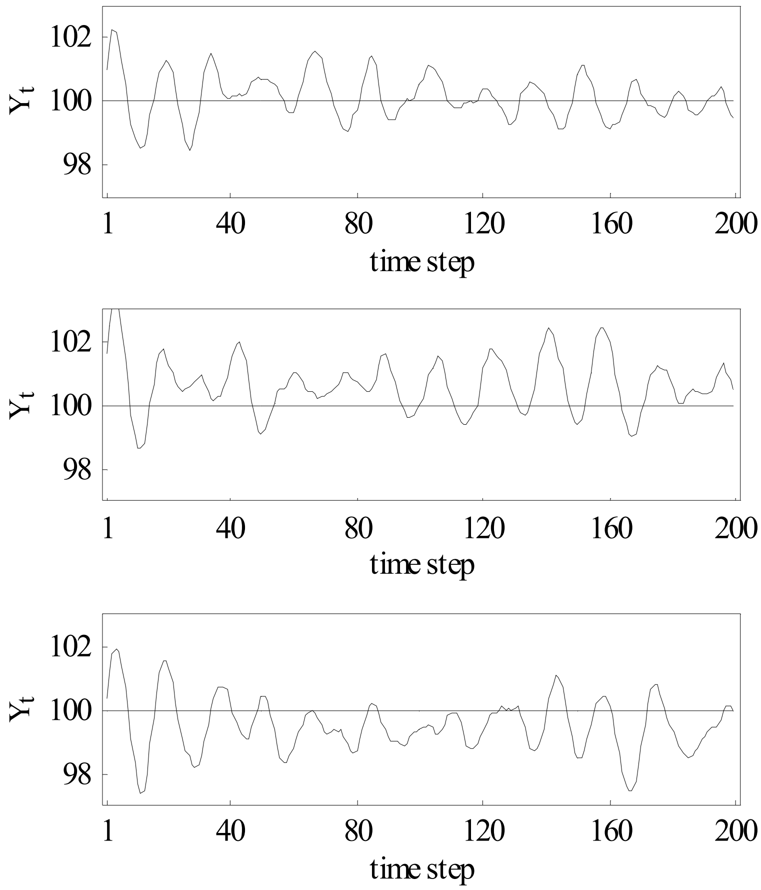

Figure 3: Random investment. The same simulation design as in figure 2, but now we add IID random shocks to the model with $\delta \sim N(0,0.1)$.

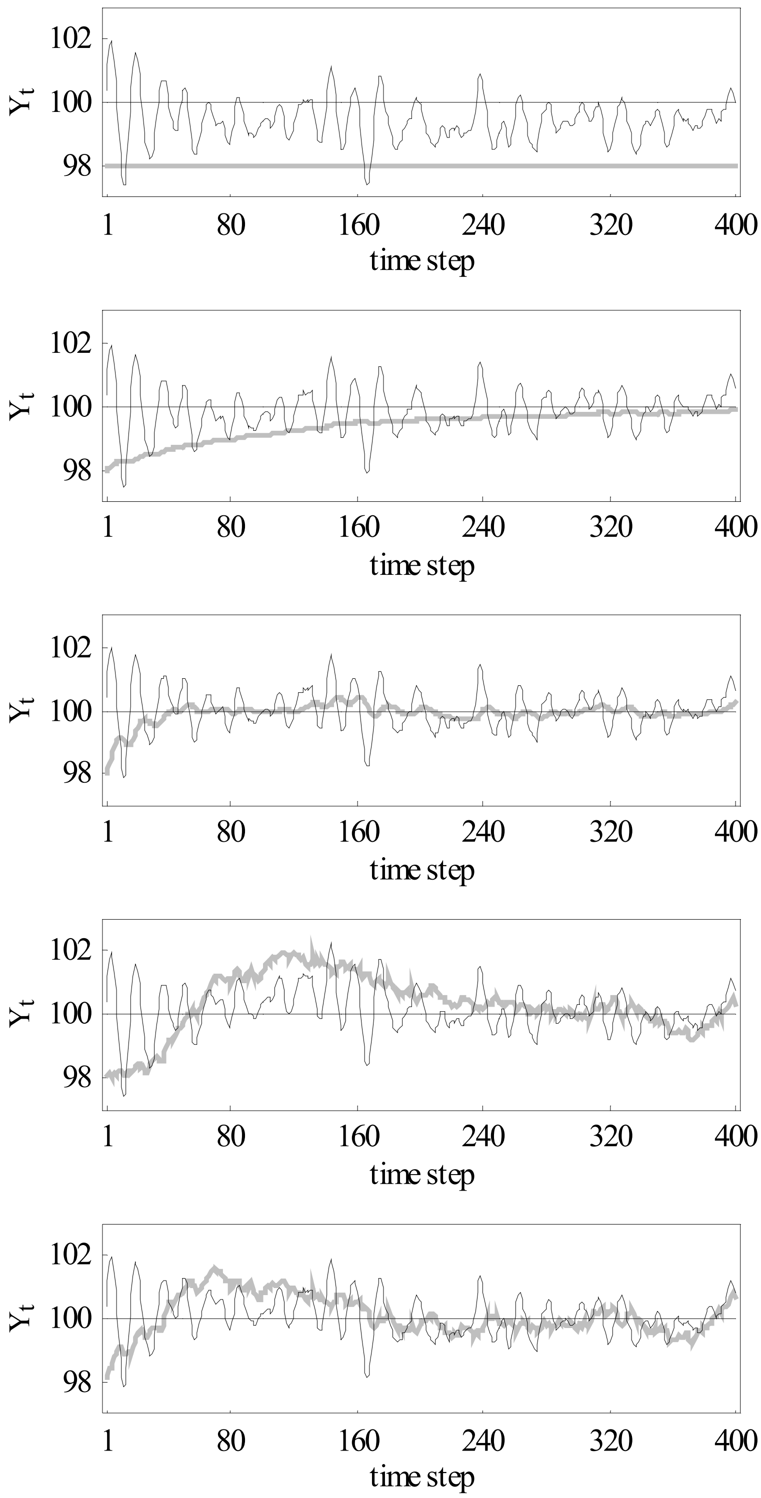

Figure 4: Perception of the "normal" level of national income (gray line). The same simulation design as in the bottom panel of figure 3 but now from top to bottom: $d = 1, \sigma^{\varepsilon} = 0$, $d = 0.99, \sigma^{\varepsilon} = 0$, $d = 0.95, \sigma^{\varepsilon} = 0$, $d = 0.99\ \sigma^{\varepsilon} = 0.1$ and $d = 0.95, \sigma^{\varepsilon} = 0.1$, respectively.